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Mutual Fund Expense Ratios:

How High is Too High?

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ABSTRACT

This paper derives a quantitative means for determining the highest expense ratio a particular mutual fund can have such that its prospective inclusion in a portfolio is expected to increase the portfolio's risk-adjusted returns. The approach provides a useful decision aid to assist in assembling portfolios of mutual funds. While the approach is most applicable to index funds, it can also be applied to non-index funds, to the extent that an index exists which is sufficiently representative of the fund's characteristics.

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Investors are often involuntarily restricted to relatively high-fee mutual funds with which to implement some of their asset allocation desires. One common example is the typical 401(k) retirement savings plan. Most plans primarily offer high fee actively managed mutual funds, often in addition to a single (lower fee) S&P 500 index fund. If an investor in that plan desires small cap stock exposure, for example, would the high fees associated with the only available small cap stock fund completely negate the anticipated increase in their portfolio's risk-adjusted return? If so, it may be more beneficial to either abandon the idea of diversifying into small cap stocks entirely, or perhaps it may be best to get exposure to small cap stocks elsewhere in one's extended portfolio (e.g., in an IRA). Even if there were no such restrictions on available investing options, there are some asset classes where there do not exist any truly low cost mutual funds. Examples include commodity futures and international small-cap stocks.

One of the principal attractions of passively managed mutual funds (e.g., index funds) is their low expense ratios. All else being equal, a fund with lower expenses must have higher performance than a similar fund with higher expenses.¹ Investors need to know how low a mutual fund's expense ratio must be in order for it to have a beneficial effect on their portfolios; they need to know how high the expense ratio can be before it completely eliminates the anticipated increase in the portfolio's expected risk-adjusted return. This question is central to the selection of mutual funds for implementation of an asset-allocation plan.

This paper attempts to answer the question: “For mutual fund expense ratios, how high is too high?”

Specifically, the paper derives an equation which answers the question, “What is the highest expense ratio I should be willing to tolerate when adding a fund to my portfolio?” The problem is solved both for the simple case (where the existing portfolio consists of one fund) and the more general case (where there are arbitrarily many funds in the existing portfolio and there are arbitrarily many changes to portfolio composition being contemplated). The approach provides a methodology for validating or denying the prudence of choosing any particular investment vehicles to implement an asset allocation plan.

Investors who are sensitive to investing expenses tend to naturally gravitate towards index funds. Therefore, unless noted otherwise, all references to mutual funds in this paper are to index funds, though the approach *may* apply equally well to actively managed funds.

The Problem

Investors are increasingly embracing the tenets of Modern Portfolio Theory (“MPT”).² Strategic asset allocators typically apply MPT principals at the asset class level when designing portfolios. They desire to build their portfolios with asset classes which have been shown to (or are expected to) have low correlations with each other. This is usually done by comparing representative indexes and modeling portfolios made up of those indexes.

Once a desired asset allocation is determined, an appropriate investment vehicle must be identified for each asset class. Portfolio optimization implies minimizing expenses when selecting each investment vehicle. For mutual funds, the principal net ongoing investing expenses are represented by the annual expense ratio,³ which pays for certain management, distribution, and administrative expenses, as well as providing profit for the mutual fund sponsor.

There are several asset classes which might be considered desirable from a diversification standpoint where virtually all available mutual funds have relatively high expense ratios.⁴ At some point, the expense ratio for a prospective mutual fund will eliminate any expected benefit from using it to diversify one’s portfolio. How high can that expense ratio be in order to still expect to realize increased risk-adjusted portfolio returns?

The Solution

In order to ensure we are actually improving the risk-adjusted performance of your portfolio, we need to have some means of measuring it. The most popular means of measuring risk-adjusted performance is the Sharpe Ratio (see Sharpe [1966] and Sharpe [1994]).⁵ In this section, the Sharpe Ratio is used to derive a solution.⁶ A more general version of the solution at the portfolio level is derived in Appendix B.

Assumptions

I assume that:

- The investor's goal is to improve their portfolio's risk-adjusted return through prospective addition of an additional mutual fund.
- A relevant index (or blend of indexes) adequately representative of each fund's respective investment style is identifiable. Note that this criterion naturally suggests that index funds are best suited to the approach described, but actively managed funds are not necessarily excluded.
- The returns and volatilities of each relevant index, as well as the correlations between them, are known. The investor can either use known values of the past or predicted values of the future, as is their preference.

- If the initial portfolio contains more than one fund, each fund is proportionally divested in order to diversify into the new fund (i.e., the relative proportions of each fund in the initial portfolio remain the same in the proposed new portfolio containing the proposed new fund). Example D illustrates this assumption well. Note that this assumption is relaxed for the more general solution derived in Appendix B.
- Costless rebalancing occurs each period when modeling returns of theoretical portfolios made up of various indexes. This only affects how the statistics used as inputs to the equation are generated. Other rebalancing frequencies can be assumed without altering the validity of the overall approach.

The Solution:

If you define the “performance differential” as the difference between a portfolio’s total return and the return of the risk-free asset, the Sharpe Ratio (as shown in Equation 1) is defined as the mean of the performance differential divided by its standard deviation.⁷

$$S = \frac{r_p - r_f}{\sigma_{p-f}} \quad (1)$$

Where r_p is the realized return on the portfolio, r_f is the risk-free rate of return,⁸ and σ_{p-f} is the standard deviation of the “performance differential” [$r_p - r_f$].

Let the initial portfolio, P , represent some existing mutual fund you already own. Let subscript n represent some other “new” fund you are considering combining with your existing fund to form a proposed new portfolio, represented by a prime (e.g., P'). You propose to have the new fund make up a percentage of your contemplated new portfolio represented by the fractional weight w .

Set a constraint that the Sharpe Ratio of the proposed new portfolio must be at least as high as the original portfolio’s Sharpe Ratio:

$$S \leq S' \tag{2}$$

By substituting Equation 1 into Equation 2, you get:

$$\left(\frac{r_p - r_f}{\sigma_{p-f}} \right) \leq \left(\frac{r'_p - r_f}{\sigma'_{p-f}} \right) \tag{3}$$

Mutual fund returns generally do not exactly match their representative indexes. There is some differential return (“DR”) representing the difference between the index’s performance and the fund’s performance:⁹

$$r_p = r_I - DR \tag{4}$$

Where r_I is the return on the index (or combination of indexes) representing the portfolio P and DR is the differential return of the associated

mutual fund (or the weighted average differential return of a portfolio of mutual funds).

Thus Equation 3 becomes:

$$\left(\frac{r_I - DR - r_f}{\sigma_{p-f}} \right) \leq \left(\frac{r'_I - DR' - r_f}{\sigma'_{p-f}} \right) \quad (5)$$

Note that the differential return of the proposed portfolio is just a weighted average of the new fund's differential return and the original fund's (or the original portfolio's weighted average) differential return:

$$DR' = w(DR_n) + (1-w)(DR) \quad (6)$$

Substituting Equation 6 into Equation 5 and multiplying by the denominators, we get:

$$\sigma'_{p-f} (r_I - DR - r_f) \leq \sigma_{p-f} (r'_I - w(DR_n) - (1-w)(DR) - r_f) \quad (7)$$

Solving for DR_n yields the following:

$$DR_n \leq \left[\frac{r'_I - (1-w)DR - r_f - \left(\frac{\sigma'_{p-f}}{\sigma_{p-f}} \right) (r_I - DR - r_f)}{w} \right] \quad (8)$$

Equation 8 shows the maximum differential return you should be willing to tolerate in a proposed new mutual fund you are considering adding to your portfolio. It is robust for all cases, supposing that the Sharpe Ratio remains a valid measure of risk-adjusted performance.

In order to relate this solution to a fund's expense ratio, a relationship between differential return and expense ratio must be determined.

Relatively little empirical research has been done in modeling index fund returns. However, the studies to date make a strong case that an index fund's differential return can be well approximated by the fund's expense ratio.¹⁰

$$DR \approx ER \tag{9}$$

Where ER is a fund's expense ratio. Substituting Equation 9 into Equation 8 gives:

$$ER_n \leq \left[\frac{r'_I - (1-w)ER - r_f - \left(\frac{\sigma'_{p-f}}{\sigma_{p-f}} \right) (r_I - ER - r_f)}{w} \right] \tag{10}$$

This equation shows the highest expense ratio that you should be willing to tolerate for the fund you are considering adding to your portfolio. If the fund being considered has an expense ratio higher than that value, the resulting portfolio would have a lower expected risk-adjusted return than the original portfolio and the prospective fund should be rejected in favor of the original portfolio or some other better alternative.

Examples

- A) Assume you currently own the Vanguard 500 Index Fund (VFINX). You are considering diversifying internationally by putting 40% of your portfolio into the Vanguard Developed Markets Index Fund (VDMIX). VFINX tracks the S&P 500 index, while VDMIX tracks the MSCI EAFE Net index. By examining Table 1, we see that you ought to reject the proposed fund because its expense ratio is so high that it would result in a reduced Sharpe Ratio for the resulting portfolio (i.e., 0.34% is greater than the maximum allowed 0.18%).
- B) Assume you currently own the Vanguard 500 Index Fund (VFINX). You are considering diversifying into smaller stocks by putting 20% of your portfolio into the Bridgeway Ultra-Small Company Market Fund (BRSIX), which roughly tracks the CRSP 10 index. The Bridgeway fund has a relatively high 0.75% annual expense ratio. Is it “worth it?” Table 1 clearly suggests that you should accept BRSIX as 20% of the new portfolio (i.e., 0.75% is less than the maximum 2.55% allowed).
- C) Assume you currently own the Vanguard 500 Index Fund (VFINX). You are considering diversifying into commodities by putting 10% of your portfolio into the Oppenheimer Real Asset Fund (QRAAX), which

roughly tracks the Goldman Sachs Commodity Index (GSCI). QRAAX has a seemingly high expense ratio of 1.68%. Is it “worth it?”

This is an example of applying the methodology to an actively managed fund. QRAAX, while it has an explicit (though non-binding) goal of having at least a 90% correlation with the GSCI, is basically an actively managed fund. However, from its inception through the end of 2002, its quarterly returns have a correlation of 98.5% with the GSCI. This fund tracks its benchmark almost as well as a true index fund. We’re confident that the GSCI is “adequately representative” of this fund’s investing style.

Table 1 shows that you should accept QRAAX as 10% of the new portfolio (i.e., 1.68% is less than the maximum 6.16% allowed).¹¹ The degree to which the fund’s expense ratio is below the maximum allowed suggests that commodities are an extremely effective diversifier.

D) This procedure works equally well if you currently own a *portfolio* of funds and you are considering adding an additional fund.

Suppose that you presently owned a portfolio consisting of 80% Vanguard 500 Index Fund (VFINX) and 20% Vanguard Developed Markets Index

Fund (VDMIX). You are considering diversifying 30% of the portfolio into the Vanguard Value Index Fund (VIVAX) (so that the resulting portfolio is 56% Vanguard 500 Index Fund, 14% Vanguard Developed Markets Index Fund, and 30% Vanguard Value Index Fund). The Vanguard Value Index Fund tracks the S&P 500/Barra Value index. Table 1 shows that the proposed fund's expense ratio is below the maximum allowed – it is “worth it” to diversify your portfolio as proposed (i.e., 0.22% is less than the maximum 1.28% allowed).

E) Some mutual funds are only available to individuals if obtained through certain financial advisors. For example, Dimensional Fund Advisors funds are distributed to the retail market in that fashion. Individuals often ask whether it is “worth it” to pay a financial advisor an additional percentage annually in order to gain access to those funds. The techniques introduced in this paper can be used to help answer that question.

Assume that your current portfolio consists of the following funds in equal percentages:

- Vanguard 500 Index Fund (VFINX); ER = 0.18%
- Vanguard Value Index Fund (VIVAX); ER = 0.22%
- Vanguard Small Cap Index Fund (NAESX); ER = 0.27%

- iShares Russell 2000 Value Index Fund (IWN);¹² ER = 0.25%

Further assume that you are considering transitioning your portfolio to the following funds, also in equal percentages:

- DFA US Large Company Portfolio (DFLCX); ER = 0.15%
- DFA US Large Cap Value Portfolio (DFLVX); ER = 0.31%
- DFA US Micro Cap Portfolio (DFSCX); ER = 0.56%
- DFA US Small Cap Value Portfolio (DFSVX); ER = 0.56%

Note that the weighted average expense ratio of the initial portfolio is 0.2225%. For the proposed new portfolio, it is 0.3950%. Since the entire portfolio is being replaced, $w = 1.0$.

Are the generally higher expense ratios of the DFA funds “worth it?” If so, how much should you be willing to pay the financial advisor?

Table 1 holds the answer. It does indeed seem “worth it” to switch to the DFA funds (i.e., 0.395% per year is less than the maximum of 1.68% per year allowed). Additionally, even if the financial advisor provides no value to the investor other than giving them access to the DFA funds (an assumption which is hopefully not valid in the majority of cases), it is still

“worth it” to pay the financial advisor as much as 1.285% of assets managed annually ($1.68\% - 0.395\% = 1.285\%$).

Discussion

The derived equations are robust and quite useful to investors to the extent that the assumptions are valid. It might be useful to examine some of those assumptions in order to assess the validity of the approach.

One assumption was that the investor’s goal is to improve their portfolio’s risk-adjusted return. Most would agree in principle with this assumption. However, the exact nature of “risk” which each investor is most sensitive to¹³ is a matter of debate and it is further not clear whether there exists a single best way to adjust performance figures to reflect that risk. The risk-adjusted return measure used here is the most widely used over the last three decades.

In Equation 9, an index fund’s expense ratio was assumed to be a good estimate of the fund’s future differential return. While this assumption is supported by what little research exists in the literature, Elton, *et al.* [2003] notes that an index fund’s past differential return may be a slightly better predictor of its future differential return than is its expense ratio.¹⁴ While that may be the case, a significantly long track record is necessary in order to provide a useful estimate of that DR. If such a long track record is available, the investor can use it to estimate a fund’s expected differential return and use Equations 8 or 11 to decide

whether the expected differential return is excessive. However, this doesn't directly answer the question of whether a fund's current expense ratio, which is more readily observable, is excessive. Further, it is an unfortunate fact that few index funds have track records at this point in time which are long enough to be analyzed in this fashion. Therefore, it may be best for investors to use the current expense ratio as a pragmatic first order estimate of future differential return.¹⁵

While there has been relatively little research in modeling index funds, there has been significantly more activity in attempting to model returns of actively managed funds. Equation 9 *may* hold equally well for actively managed funds: there is strong evidence that there exists a significant negative correlation between expense ratio and performance for actively managed funds. Carhart [1997] estimated that, for every 100 basis points of expense ratio, performance decreased by 153 basis points. Malkiel [1995] found that 100 basis points of expense ratio decreased performance by 192 basis points, but that the coefficient was not significantly different from -1.0. For bond funds, the coefficient appears to be almost exactly -1.0 (i.e., 100 bp of expense ratio results in 100 bp of reduced performance).¹⁶ When applying this paper's techniques to actively managed funds, it is crucial to identify an index (or a blend of indexes) that is adequately representative of the active fund's investing strategy. If a fund exhibits significant style drift over time (as actively managed funds tend to do), the results of this paper's analyses progressively lose significance as the fund's style drifts

further from that assumed during the analysis. The fact that style drift can be completely avoided with an index fund is one reason why the investor might prefer to avoid actively managed funds altogether.

Assuming Past is Prologue

One of the assumptions in our approach was that performance and volatility values for relevant indexes are known. This is obviously only true for *past* values. Perfect knowledge of past performance in general only guarantees a portfolio optimized for past conditions (see Ang, Chua, and Desai [1980]). However, most investors are more interested in optimizing a portfolio's *future* performance. In order for the approach described herein to be useful for optimizing a portfolio's future performance, one or both of the following must be true:

- Past index performance, volatility, and correlations must be at least *somewhat representative* of the index's future behavior; or
- Future index performance, volatility, and correlations can be estimated with some accuracy.

In general, the second condition is only true to the extent that the first is true. The first condition is only true over long periods of time and even then, the persistence of index returns, volatility, and correlations is noisy, at best. If a sufficiently long data series for each of the indexes in question is not available, the techniques

suggested in this paper are of little use unless some means of accurately forecasting index performance in the future is available.¹⁷

This criticism applies not only to our treatment in this paper, but to *any* analysis which uses the Sharpe Ratio in order to draw conclusions about what sort of investment activity might be prudent in the future. Indeed, even the famous Black-Scholes formula for option valuation (see Black and Scholes [1973]) requires an estimate of a security's future volatility. If the Black-Scholes formula has utility despite imperfect knowledge of future volatility (and most would agree that it does), then this paper's approach might similarly have utility, despite its dependence on estimated performance parameters.

To the extent that the measurable past (or the otherwise predictable future) is a relevant predictor of the future, the approach described herein provides a decision aid which is useful for portfolio design and implementation. *As imperfect as it may be*, the approach is an improvement on the alternative, which is simply *guessing* whether or not a particular fund's expense ratio will detract from the fund's performance so much as to outweigh the expected benefits of including that fund in a portfolio.

Conclusions and Suggestions for Further Research

Investors are increasingly turning to index mutual funds to implement their asset allocation plans. This is often done in order to realize expected

benefits of diversification with other elements of their portfolios. Such investors are often conflicted between the benefits of diversification promised (by adding a particular asset class to their portfolio) and a reluctance to pay the seemingly high expense ratios that sometimes accompany available index funds which invest in those asset classes. This paper provides a robust tool for assisting investors in resolving that dilemma. The tool's utilization depends on knowledge of the relationship of an index fund's differential return and its expense ratio. A first order estimate of this relationship was presented and used for examples. Additional research in modeling index fund performance is clearly warranted. Improved models of the relationship between an index fund's expense ratio and its differential return will further increase the utility of the approach suggested.

Notes

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¹ See Sharpe [1991].

² See Markowitz [1952].

³ There are other fees, such as front or back end sales loads, internal transaction expenses, etc. which also generally ought to be minimized, but a detailed discussion thereof is beyond the scope of this paper. For more information on the relationship of mutual fund transaction expenses and performance, see Chalmers, Edelen, and Kadlec [2001].

⁴ Examples include, but are not limited to: Commodity Futures, Small Cap Stocks, Emerging Market Stocks, Global Bonds, etc.

⁵ There exist several other viable measures of risk-adjusted performance. The most often used alternatives to the Sharpe Ratio are Jensen's Alpha and the Treynor Performance Index. However, not all measures are amenable to use in the manner demonstrated in this paper. For example, the Upside-Potential Ratio is one such viable measure which is not well suited for this treatment.

⁶ Though unreported here, Jensen's Alpha and the Treynor Performance Index were used to derive conceptually equivalent solutions. For the sake of brevity and clarity, those derivations aren't included here. In empirical tests also unreported here, both the Jensen and Treynor versions of the solution were less limiting (i.e., less conservative) than the Sharpe version.

⁷ Note that the formula used here for the Sharpe Ratio differs somewhat from that commonly used by others. Many others use σ_p , the standard deviation of the portfolio, as the denominator. We use the definition advocated by Sharpe — it gives a performance measure adjusted for the volatility of the portfolio performance differential (i.e., the volatility of the “risky” component of the return). See Sharpe [1994] for a more elaborate discussion of related issues. In practice, using either σ_p or σ_{p-f} usually yields very similar results.

⁸ One month Treasuries are used as the “risk-free” measure in the examples.

⁹ Note that this paper's definition of “differential return” is similar to what is often referred to as “tracking error.” However, some might define differential return (or tracking error) as the negative of this definition. The definition used here is useful because it somewhat simplifies the derivation.

¹⁰ See Frino and Gallagher [2001]: “As expected, index funds earn significantly negative raw and risk-adjusted excess returns, and the margin of

underperformance is roughly equivalent to the average expense ratio.” Also, see Elton, Gruber, and Busse [2003]: “... differential return has a very high R^2 with past expenses (0.768). The relationship is significant at the one percent level. Furthermore, expenses on average lower differential return by the amount of the expenses, since differential return goes down by 0.999 percent for every one percent increase in expenses.” Note that Elton, *et al.*'s [2003] definition of differential return is the negative of this paper's definition. So, by our definition, differential return would increase with increases in expenses.

¹¹ Note that this analysis ignores the initial sales commission of 5.75%. If the fund's expense ratio were adjusted upward to reflect the sales commission (perhaps by allocating it across a typical anticipated holding period of perhaps five years), the fund would still be “worth it” in the example.

¹² This is actually an Exchange Traded Fund (“ETF”), which is somewhat different from the open-ended mutual funds otherwise discussed here. This paper's approach should apply equally well to ETFs. This ETF was used in the example instead of the similar Vanguard Small Cap Value Index Fund (VISVX) due to the fact that the Russell 2000 Value index (which IWN tracks) has a dramatically longer history (beginning in 1979) than the S&P/Barra Small Cap 600 Value index (which VISVX tracks, but starts as recently as 1994). This illustrates a limitation of the approach described in this paper: it requires indexes

to have adequately long histories, unless future values can somehow otherwise be estimated.

¹³ For example, the Sharpe Ratio uses standard deviation of the performance differential as a risk-proxy, while the Jensen and Treynor measures use beta. An excellent alternative measure of risk is downside risk – See Van Harlow [1992].

¹⁴ “... the coefficient of determination increases to 0.845 when we substitute past differential return for past expenses. Investors interested in differential return can apparently choose an index fund simply by looking at the past expense ratio, but can do even better by looking at past differential return.”

¹⁵ Note that Elton, *et al.* [2003] showed that expense ratios tend to persist: “... past expenses are almost perfect predictors of future expenses. We demonstrate the stability of expenses by noting that the coefficient of determination between past and future expenses is 0.931 with a slope of 0.997.” Using expense ratio instead of past differential return as a predictor of future differential return has the desirable effect of making our analysis completely independent of any particular fund’s past performance. This makes the calculation easier and it makes the approach feasible for many relatively new funds, supposing that their corresponding indexes have adequately long operating histories.

¹⁶ See Blake, Elton, and Gruber [1993], Domian and Reichenstein [1997],
Domian and Reichenstein [2002], and Reichenstein [1999].

¹⁷ For example, if an index that a particular fund is based on only has a history of
two years, this paper's approach probably cannot be used with any degree of
confidence. The longer the index data series, the better.

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Appendix A

Data Sources

- Russell 2000 index provided courtesy of Dimensional Fund Advisors.
- Russell 2000 Value index provided courtesy of Dimensional Fund Advisors.
- S&P 500 index provided courtesy of Barra, Inc.
- S&P 500/Barra Value index provided courtesy of Barra, Inc.
- CRSP 9-10 index provided courtesy of Dimensional Fund Advisors.
- CRSP 10 index provided courtesy of Dimensional Fund Advisors.
- MSCI EAFE Net index provided courtesy of Morgan Stanley Capital International.
- Fama/French Large Cap Value benchmark index provided courtesy of Kenneth French.
- Fama/French Small Cap Value benchmark index provided courtesy of Kenneth French.
- Goldman Sachs Commodities Index – Total Return provided courtesy of Campbell Harvey.
- One month Treasuries returns provided courtesy of Dimensional Fund Advisors.

Appendix B

More Generalized Solution for Multi-Fund Portfolios

If you relax the assumption that the funds in the initial portfolio remain in the same relative proportions in the proposed new portfolio (i.e., you allow the proposed portfolio's composition to be completely independent of the initial portfolio's), it is possible to develop a more general (albeit somewhat more complex) solution at the portfolio level (i.e., to better address situations like Example E).

Suppose that the initial portfolio can contain any number of funds, each with its weight varying from 0% to 100% of the portfolio. Their relative proportions need not remain fixed in the proposed new portfolio. You wish to judge whether the *weighted average* expense ratio of a proposed new portfolio is “worth it,” given the present composition of your portfolio and the proposed composition of the new portfolio. It can be shown that the Sharpe Ratio version of the general solution is:

$$\sum_{i=1}^m (w'_i DR_i) \leq \left[r'_I - r_f - \left(\frac{\sigma'_{p-f}}{\sigma_{p-f}} \right) \left(r_I - r_f - \sum_{i=1}^m (w_i DR_i) \right) \right] \quad (11)$$

Where m is the number of mutual funds in the mutual fund universe being considered, w_i is the fraction of the initial portfolio invested in the i^{th} mutual fund, w'_i is the fraction of the proposed new portfolio to be invested in the i^{th} mutual fund, and DR_i is the differential return of the i^{th} mutual fund

Substituting in Equation 9 gives:

$$\sum_{i=1}^m (w'_i ER_i) \leq \left[r'_I - r_f - \left(\frac{\sigma'_{p-f}}{\sigma_{p-f}} \right) \left(r_I - r_f - \sum_{i=1}^m (w_i ER_i) \right) \right] \quad (12)$$

Where ER_i is the expense ratio of the i^{th} mutual fund.

Equation 11 is robust for all cases. Equation 12 depends on the validity of Equation 9.

Table 1, Examples

	A	B	C	D	E
Existing Mutual Fund	Vanguard 500 Index Fund (VFINX)	Vanguard 500 Index Fund (VFINX)	Vanguard 500 Index Fund (VFINX)	80% VFINX and 20% VDMIX	VFINX, VIVAX, NAESX, and IWN in equal proportions
Proposed New Mutual Fund	Vanguard Developed Markets Index Fund (VDMIX)	Bridgeway Ultra-Small Company Market Fund (BRSIX)	Oppenheimer Real Asset Fund (QRAAX)	Vanguard Value Index Fund (VIVAX)	DFLCX, DFLVX, DFSCX, DFSVX in equal proportions
ER_n, Expense Ratio of Existing Fund	0.015% per mo. (0.18% per year)	0.015% per mo. (0.18% per year)	0.015% per mo. (0.18% per year)	0.018% per mo. (0.212% per year)	0.019% per mo. (0.2225% per yr)
σ_{p-f}, Std Dev of initial performance differential	4.58% per month	5.66% per month	4.58% per month	4.22% per month	4.57% per month
σ'_{p-f}, Std Dev of proposed new performance differential	4.16% per month	6.30% per month	4.14% per month	4.20% per month	4.50% per month
r_I, Mean return of initial fund's index	0.964% per mo.	0.971% per mo.	0.964% per mo.	1.119% per mo.	1.171% per mo.
r'_I, Mean return of proposed new portfolio using representative indexes for each represented fund (assuming costless rebalancing each period)	0.924% per month	1.084% per month	0.973% per mo.	1.143% per month	1.282% per month
r_f, Mean return of risk-free asset (one month Treasuries)	0.521% per month	0.310% per month	0.521% per month	0.528% per month	0.537% per month
w, fraction of proposed new portfolio dedicated to proposed new index fund	0.4	0.2	0.1	0.3	1.0
Maximum ER_n	0.015% per mo. (0.18% per year)	0.212% per mo. (2.55% per year)	0.513% per mo. (6.16% per year)	0.106% per mo. (1.28% per year)	0.140% per mo. (1.68% per year)
Actual ER_n for proposed new mutual fund	0.028% per mo. (0.34% per year)	0.063% per mo. (0.75% per year)	0.14% per mo. (1.68% per year)	0.018% per mo. (0.22% per year)	0.033% per mo. (0.395% per year)
Accept/Reject proposed new mutual fund?	Reject; Actual ER_n is too high	Accept; Actual ER_n is below maximum	Accept; Actual ER_n is below maximum	Accept; Actual ER_n is below maximum	Accept; Actual ER_n is below maximum
Data Period Used	1970-2002	1926-2002	1970-2002	1975-2002	1979-2002